A Marketing-Oriented Inventory Model with Three-Component Demand Rate and Time-Dependent Partial Backlogging

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Abstract— This paper, an attempt has been made to extend the model of "An EOQ model for perishable items under stock-dependent selling rate and time-dependent partial backlogging" with a view to making the model more flexible, realistic and applicable in practice. Here, objectives are to maximize the profit and minimize the total shortage cost. In this model, fuzzy goals are used by linear membership functions and after fuzzification, it is solved by weighted fuzzy non-linear programming technique. The model is illustrated with a numerical example adopted partially from "An EOQ model for perishable items under stock-dependent selling rate and time-dependent partial backlogging".

Keywords— EOQ; Perishable items; Partial back logging; Fuzzification; Membership function.

I. INTRODUCTION

In the competitive market situation, it is commonly observed that an increase in shelf space and glamorous display for an item induce more consumers to buy it. Recently, Dye and Ouyang (2005) investigated an economic order quantity (EOQ) model for perishable items under stock-dependent selling rate and timedependent partial backlogging. In two-component demand, it is assumed that the demand rate is stockdependent down to a certain level and then it becomes constant. But, it is commonly observed that the demand rate will not be dependent on displayed stock level for a huge amount of stock as all available stock cannot be displayed properly and glamorously because of cost of modern light, electronic arrangement and space will be increased (e.g. fashionable goods shop). It will be dependent on displayed stock level within a range and beyond this range, it will be quite uniform. This type of demand rate is called three-component demand rate.

It has been recognized that one's ability to make precise statement concerning an inventory model diminishes with increasing complexities of the system. Generally, it may not be possible to define the objective goals precisely. In reality, management is most likely to be uncertain of the true value of parameters and due to many unforeseen incidents like strike, hike in wages, increased transportation cost etc; hence during the course of business, a vendor or decision maker is forced to settle down with a lower profit amount compared to the profit as he/she normally has targeted due to adverse situation. Moreover, shortages bring loss of goodwill for the vendor. This loss can not be measured numerically. For this reason, it is advisable to restrict the shortages as much as possible to minimize the loss of goodwill. From the above discussion, we may conclude that it is difficult to determine the exact amount of profit and shortage cost rather a range may be fixed for these. Hence, under these phenomena the inventory model may be better treated in a fuzzy system.

II. NOTATIONS AND MODELING ASSUMPTIONS

In this section, we give the notations and assumptions used throughout this chapter.

- 2.1 The inventory system involves only one item.
- 2.2 Replenishment rate is infinite and lead time is zero.
- 2.3 θ , constant rate of deterioration. I(t) is the inventory level at time t (Fig. 1).
- 2.4 p, the selling price per unit and A, the ordering cost per order, are constant.
- 2.5 The unit cost C and the inventory carrying cost as fraction i, per unit per unit time, are constant.
- 2.6 Shortages are allowed and backlogged rate is defined to be $1/[1+ \delta(T-t)]$. The backlogging parameter δ is a positive constant. Shortage cost is C₂ per unit per unit time and R is the fixed opportunity cost of lost sales per unit.
- 2.7 The demand rate D(p, I(t)), is dependent on selling price and displayed stock level in the show room with-in the stock level S_0 to S_1 and beyond this range, it becomes constant with respect to the display stock level. The functional D(p, I(t)), is given by:

$$\begin{aligned} \alpha & (p) + \beta S_1 \\ \alpha & (p) + \beta I(t) \\ D(p, I(t)) &= \alpha & (p) \\ \frac{\alpha(p)}{1 + \delta(T-t)} I(t) &\leq 0 \end{aligned} \quad \begin{array}{l} I(t) &\geq S_1, \\ S_0 &\leq I(t) \leq S_1, \\ 0 &\leq I(t) \leq S_0, \\ I(t) &\leq 0 \end{aligned}$$

Where, β is a non-negative constant. α (p) is a non-negative function of selling price p.

- 2.8 Shortages are allowed and backlogged rate is defined to be $1/[1+\delta(T-t)]$. The backlogging parameter δ is a positive constant. Shortage cost is C₂ per unit per unit time and R is the fixed opportunity cost of lost sales per unit. 2.9 T is the cycle time.
 - 2.10TP and SC respectively denote the total

III. THE MATHEMATICAL MODEL

At the beginning of the order cycle the inventory level is raised to Q afterwards as time progresses it is depleted by combined effects of the demand and deterioration. The pictorial representation of the inventory system is given in Fig. 1. Therefore, the differential equations governing the system during the period ($0 \le t \le T$) can be written as:

$$\begin{split} \frac{dI(t)}{dt} + \theta I(t) &= -(\alpha(p) + \beta S_1), \\ I(t) &\geq S_{1,} 0 \leq t \leq t_1 \quad (1) \\ \frac{dI(t)}{dt} + \theta I(t) &= -(\alpha(p) + \beta I_1(t)), \\ S_0 &\leq I(t) \leq S_1, t_1 \leq t \leq t_2 (2) \\ \frac{dI(t)}{dt} + \theta I(t) &= -\alpha(p), \\ 0 &\leq I(t) \leq S_0, t_2 \leq t \leq t_3 \quad (3) \\ \frac{dI(t)}{dt} &= -\frac{\alpha(p)}{1 + \delta(T - t)}, \end{split}$$

$$I(t) \le 0, \quad t_3 \le t \le T \quad (4)$$

The solutions of the above differential equations, after applying boundary conditions $I(t_1) = S_0$, $I(t_2) = S_1$, $I(t_3) = 0$, are

$$\begin{split} \mathbf{I}(t) &= \mathbf{S}_{1} \mathbf{e}^{\theta \begin{pmatrix} t_{1} - t \end{pmatrix}} + \frac{\left(\alpha(\mathbf{p}) + \beta \mathbf{S}_{1}\right)}{\theta} \left(\mathbf{e}^{\theta \begin{pmatrix} t_{1} - t \end{pmatrix}} - \mathbf{I} \right) \\ 0 &\leq t \leq t_{1} \qquad (5) \\ \mathbf{I}(t) &= \mathbf{S}_{0} \mathbf{e}^{(\theta + \beta)(t_{2} - t)} + \frac{\alpha(\mathbf{p})}{(\theta + \beta)} \left(\mathbf{e}^{(\theta + \beta)(t_{2} - t)} - \mathbf{I} \right), \\ t_{1} &\leq t \leq t_{2} \qquad (6) \\ \mathbf{I}(t) &= \frac{\alpha(\mathbf{p})}{\theta} \left(\mathbf{e}^{\theta (t_{3} - t)} - \mathbf{I} \right), \\ t_{2} &\leq t \leq t_{3} \qquad (7) \\ \mathbf{I}(t) &= \frac{\alpha(\mathbf{p})}{\delta} \left\{ -\ln|\mathbf{I} + \delta(\mathbf{T} - t_{3})| + \ln|\mathbf{I} + \delta(\mathbf{T} - t)| \right\} \\ t_{3} &\leq t \leq T \qquad (8) \end{split}$$

Ordering cost per cycle = A Holding cost per cycle =

$$\operatorname{Ci}\left[\int_{0}^{t_{1}} \mathbf{I}(t) dt + \int_{t_{1}}^{t_{2}} \mathbf{I}(t) dt + \int_{t_{2}}^{t_{3}} \mathbf{I}(t) dt\right]$$

Shortage cost per cycle (SC) =

$$C_{2}\int_{t_{3}}^{T}\frac{\alpha(p)}{\delta}\left\{\ln\left|1+\delta(T-t_{3})\right|-\ln\left|1+\delta(T-t)\right|\right\}dt$$

Opportunity cost due to lost sales per cycle =

$$\alpha(p) R \int_{t_3}^{T} \left\{ 1 - \frac{1}{1 + \delta(T - t)} \right\} dt$$

Purchase cost

$$= C \left\{ \frac{\alpha(p)}{\delta} \ln \left| 1 + \delta (T - t_3) \right| + S_1 e^{\theta t_1} + \frac{(\alpha(p) + \beta S_1)}{\theta} \left(e^{\theta t_1} - 1 \right) \right\}$$

Sales revenue per cycle =

$$S \begin{bmatrix} t_1 \\ 0 \\ 0 \end{bmatrix} (\alpha(p) + \beta S_1) dt + \int_{t_1}^{t_2} (\alpha(p) + \beta I(t)) dt + \int_{t_2}^{t_3} (\alpha(p) + \beta S_1) dt + \int_{t_3}^{T} \frac{\alpha(p)}{1 + \delta(T - t_3)} dt \end{bmatrix}$$

on integration and simplification of the relevant costs mentioned above, the total profit per unit time TP becomes, TP=

$$\begin{split} & \left[\frac{\alpha(\mathbf{p})}{\delta}(\mathbf{p}-\mathbf{C})\ln\left|1+\delta(\mathbf{T}-\mathbf{t}_{3})\right|-\frac{\alpha(\mathbf{p})}{\delta^{2}}(\mathbf{C}_{2}+\mathbf{R}\delta)\times\right.\\ & \left(\delta(\mathbf{T}-\mathbf{t}_{3})-\ln\left|1+\delta(\mathbf{T}-\mathbf{t}_{3})\right|\right)\\ & \left.+\left(\beta\mathbf{S}-\mathbf{C}\mathbf{i}\right)_{\mathbf{e}}^{\mathbf{b}}\left(^{\left(\theta+\beta\right)\left(\mathbf{t}_{2}-\mathbf{t}_{1}\right)}-1\right]\right\}\times\\ & \left\{\frac{\mathbf{S}_{0}}{\theta+\beta}+\frac{\alpha(\mathbf{p})}{\left(\theta+\beta\right)^{2}}\right\}-\frac{\mathbf{C}\mathbf{i}\mathbf{S}_{1}}{\theta}\left(\mathbf{e}^{\theta t_{1}}-1\right)+\mathbf{p}\left(\alpha(\mathbf{p})\mathbf{t}_{3}+\beta\mathbf{S}_{1}\mathbf{t}_{1}\right)\right.\\ & \left.-\mathbf{C}\mathbf{i}\frac{\alpha(\mathbf{p})}{\theta^{2}}\left\{\mathbf{e}^{\theta(t_{3}-t_{1})}-\theta(\mathbf{t}_{3}-\mathbf{t}_{2})-1\right\}-\frac{\mathbf{C}\mathbf{i}\left(\alpha(\mathbf{p})+\beta\mathbf{S}_{1}\right)}{\theta^{2}}\times\right.\\ & \left.\left(\mathbf{e}^{\theta t_{1}}-\theta \mathbf{t}_{1}-1\right)-\mathbf{A}+\frac{\alpha(\mathbf{p})(\mathbf{p}\beta-\mathbf{C}\mathbf{i})}{\left(\theta+\beta\right)}(\mathbf{t}_{1}-\mathbf{t}_{2}\right)\right.\\ & \left.-\mathbf{C}\left\{\mathbf{S}_{1}\mathbf{e}^{\theta t_{1}}+\frac{\alpha(\mathbf{p})+\beta\mathbf{S}_{1}}{\theta}\left(\mathbf{e}^{\theta t}-1\right)\right\}\right]\right/\mathbf{T},\quad(9) \end{split}$$

and total shortage cost per unit time,

$$SC = C_2 \int_{t_3}^{T} \frac{\alpha(\mathbf{p})}{\delta} \left\{ \ln \left| 1 + \delta(\mathbf{T} - t_3) \right| - \ln \left| 1 + \delta(\mathbf{T} - t) \right| \right\} dt / T$$
(10)

where S_0 and S_1 is given by,

$$\begin{split} \mathbf{S}_{1} &= -\frac{\alpha(\mathbf{p})}{(\theta+\beta)} + \left(\mathbf{S}_{0}^{(6)} + \frac{\alpha(\mathbf{p})}{(\theta+\beta)}\right) \mathbf{e}^{(\theta+\beta)(t_{2}-t_{1})} , \quad (11)\\ \mathbf{S}_{0} &= -\frac{\alpha(\mathbf{p})}{\theta} + \frac{\alpha(\mathbf{p})}{\theta} \mathbf{e}^{\theta(t_{3}-t_{2})} , \quad (12) \end{split}$$

Now from (11) and (12) we get,

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(15)

(16)

$$t_{2} = t_{1} + \frac{1}{(\theta + \beta)} \ln \left[\frac{S_{1} + \frac{\alpha(p)}{(\theta + \beta)}}{S_{0} + \frac{\alpha(p)}{(\theta + \beta)}} \right], \quad (13)$$
$$t_{3} = t_{2} + \frac{1}{\theta} \ln \left[\frac{S_{0} + \frac{\alpha(p)}{\theta}}{\frac{\alpha(p)}{\theta}} \right], \quad (14)$$

The above two equations implies

 $t_2 - t_1 > 0$,

and $t_3 - t_2 > 0$,

and the initial lot size

$$Q = I(0) = S_1 e^{\theta t_1} + \frac{(\alpha(p) + \beta S_1)}{\theta} \left(e^{\theta t_1} - 1 \right),$$
(17)

Replacing t_1 by 0 first, substitute t_2 and t_3 by t_1 , we can observed that the above profit function will be same as the profit function of Dye and Ouyang (2005).

5.1. Crips model

In crisp environment multi-objective problem of maximizing total profit and minimizing the total shortage cost can be written as follows:

Max TP Min SC

$$\begin{array}{c} \text{Subject to,}\\ t_2-t_1 \! > \! 0\\ t_3-t_2 \! > \! 0 \end{array}$$
 where $t_1,t_2,t_3,T \geq 0. \end{array} \tag{18}$

5.2Fuzzy model

Since seller's maximum average revenue and minimum total shortage cost per unit time becomes imprecise in nature, the above model in fuzzy sense can be represented as:

Max TP Min SC Subject to, $t_2 - t_1 > 0$ $t_3 - t_2 > 0$ where $t_1, t_2, t_3, T \ge 0.$ (19)

5.3 Fuzzy goal programming of model

The fuzzy multi-objective problem can be formulated as a FNLGP as follows:

Find $(t_1, t_2, t_3, T)^T$ subject to the constraints $f_1(t_1, t_2, t_3, T) = -TP \le -f_{01}$ $f_2(t_3, T) = SC \le f_{02}$ $t_2 - t_1 > 0$ $t_3 - t_2 > 0$ where $t_1, t_2, t_3, T \ge 0$.

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Here, the fuzzy goal of objectives, i.e. total average profit and total shortage cost, are $(f_{01}-P_{01}, f_{01})$ and $(f_{02}, f_{02}+P_{02})$ respectively, and there linear MFs are consider as follows:

$$\begin{array}{c} 0, \text{for} \quad f_{1}(t_{1}, t_{2}, t_{3}, T) \leq -f_{01} + P_{01} \\ \mu_{1}(f_{1}(t_{1}, t_{2}, t_{3}, T)) = \begin{cases} 1 - \frac{f_{1}(t_{1}, t_{2}, t_{3}, T) + f_{01}}{P_{01}}, \\ 1 - \frac{f_{1}(t_{1}, t_{2}, t_{3}, T) + f_{01}}{P_{01}}, \\ T \leq -f_{01} + P_{01} \\ for \quad f_{1}(t_{1}, t_{2}, t_{3}, T) \leq -f_{01} \\ 1, \text{ for } f_{1}(t_{1}, t_{2}, t_{3}, T) \leq -f_{01} \\ 1, \text{ for } TP \leq f_{01} - P_{01} \\ 1, \text{ for } TP \geq f_{01} \\ \text{and} \\ \mu_{2}(SC) = \begin{cases} 0, \quad \text{for } SC \geq f_{02} + P_{02} \\ 1 - \frac{SC - f_{02}}{P_{02}}, \text{for } f_{02} \leq SC \leq f_{02} + P_{02} \\ 1, \quad \text{for } SC \leq f_{01} \end{cases} \end{cases}$$

Using the weights to represent different importance for the objectives, the problem can be written as follows:

 $Max F = w_1\mu_1(TP) + w_2\mu_2(SC)$

 $\begin{array}{l} \text{Subject to} \\ f_{01}\text{-}P_{01} {\leq} \; \mu_1(TP) {\leq} \; f_{01} \\ f_{02} {\leq} \; \mu_2(SC) {\leq} \; f_{02} {+} P_{02} \\ t_2 {-} \; t_1 {>} \; 0 \\ t_3 {-} \; t_2 {>} 0 \\ w_1 {+} \; w_2 {=} \; 1 \\ \text{where } \; t_1, \; t_2, \; t_3, \; T {\geq} \; 0. \end{array}$

IV. NUMERICAL EXAMPLES

To illustrate the above inventory models, values of the system parameters are considered as:

A = 250.0, β = 0.3, θ = 0.08, C = 5.0, i = 0.35, C₂=3.0, p=7.0, R=5.0, δ = 10, α (p)=K(p)^{-r}, K = 20000.0, p=7.0,S₀ = 100.0, S₁= 300.0, r = 1.5, f₀₁ = -\$750.0, f₀₂ = \$30.0, P₀₁ = -\$625.0, P₀₂ = \$20.0.

The optimal values of t_1 , t_2 , t_3 , t_4 along with total profit, total shortage cost and lot-size are displayed below:

From Table-1 and Table-2, it is observed that when a seller takes care of his profit only, the seller makes maximum revenue at the cost of his reputation and goodwill. Similarly when the seller only takes care of his shortage cost, his total revenue is lower. As expected, when interests of both seller's total revenue and shortage cost are considered, then total revenue and shortage costs become moderate, i.e. it lies between the above mentioned levels.



Fig. 1. Graphical representation of inventory system

Circuit II Frank						
Crisp model	Equal	First priority	First			
	weight for	for profit	priority			
	profit &		for			
	shortage		shortage			
	cost		cost			
t1	0.2489	0.2531	0.2613			
t ₂	0.5214	0.5310	0.5124			
t ₃	0.7025	0.7112	0.7124			
Т	0.9678	0.9852	0.9675			
Profit (\$)	709.62	717.39	693.71			
Shortage cost	58.29	63.69	54.20			
(\$)						
Lot-size	537.81	542.05	550.39			

Table 1: Results for Crisp model

Table 2.	Results	for	Fuzzv	model

Fuzzy model	Equal	First priority	First
	weight for	for profit	priority
	profit &		for
	shortage		shortage
	cost		cost
t_1	0.2521	0.2641	0.3196
t ₂	0.5247	0.5482	0.5772
t ₃	0.7288	0.7441	0.7738
Т	0.9558	0.9885	0.9435
Profit (\$)	819.51	825.38	744.12
Shortage cost	43.59	50.04	25.17
(\$)			
Lot-size	543.05	533.22	610.36

VI. CONCLUSION

A multi-objective inventory model of deteriorating item with stock and price dependent demand, with shortages is developed. Here a real-life inventory problem faced by the inventory practitioners is considered. The purpose of this chapter is to investigate an inventory model for deteriorating item with three-component demand rate; permitting shortage and time-proportional backlogging rate within the economic order quantity (EOQ) framework. In the existing model Dye and Ouyang (2005), authors considered the demand rate dependent on the current displayed stock, i.e. the demand rate will be high and high for more and more displayed stock in the showroom. This is somehow unrealistic. The stock dependency nature must occur within a range, and beyond this range it will be quite uniform. Selling price is also an influencing factor on demand. Under fire over various financial ethical issues globally, some attention must be need to the replenishment cost so that it becomes minimum along with the maximum profit. Such a realistic problem has been modeled and solved under crisp and fuzzy environment. Since the proposed model has been formulated with imprecise informations, the decision maker may choose that solution which suits him/her best respect to conditions and restrictions. Till now, only a very few researchers have considered such a realistic phenomenon, though several papers dealing with an EOQ model with deterioration and time-dependent partial backlogging are available.

The scope of application of the model in supermarkets is open however, success depends on correctness of the estimation of input parameters. To estimate the parameters, demands of the same kind product in different supermarkets have to be observed and analyzed over long time.

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